

TH-3 -ENGINEERING MATHEMATICS -II

Full Marks: 80

Time- 3 Hrs

Answer any five Questions including Q No.1& 2
 Figures in the right hand margin indicates marks

1. Answer **All** questions 2 x 10
1. a. Define Modulus Function and represent it graphically.
- b. Evaluate $\lim_{x \rightarrow 0} \frac{x}{\sqrt{1+x} - \sqrt{1-x}}$
- c. Differentiate $\sec^{-1} \left(\frac{\sqrt{a^2+x^2}}{a} \right)$ with respect to x .
- d. Define unit vector and find the unit vector of the given vector $2\hat{i} + 3\hat{j} + 6\hat{k}$.
- e. Evaluate the integral $\int (e^{5 \ln x} - e^{4 \ln x}) dx$.
- f. Define Homogeneous Function and State Euler's Theorem.
- g. Find the value of α so that $\vec{a} = \hat{i} + \hat{j} + \alpha\hat{k}$, $\vec{b} = 4\hat{i} - 3\hat{k}$ are perpendicular to each other.
- h. Find the order and degree of the following differential equation

$$\frac{d^2y}{dx^2} = \frac{3y + \frac{dy}{dx}}{\sqrt{\frac{d^2y}{dx^2}}}$$
- i. Find the value of $\int_{-2}^2 |x| dx$.
- j. If $y = t^2$ and $x = t^3$ find $\frac{dy}{dx}$ at $t = 1$. 6 x 5
2. Answer **Any six** questions:
- a. If $f(x) = \begin{cases} ax^2 + b, & \text{if } x < 1 \\ 1, & \text{if } x = 1 \\ 2ax - b, & \text{if } x > 1 \end{cases}$ is continuous at $x = 1$, then find the value of 'a' and 'b'.
- b. Find $\frac{dy}{dx}$ if $y = (\ln x)^{\tan x}$.
- c. Determine the area within the curve $y^2 = 4ax$ and the x-axis, the ordinate $x=4$.
- d. Evaluate $\int \frac{\tan x + \tan \alpha}{\tan x - \tan \alpha} dx$.
- e. Solve $(1 + x^2)dy + (1 + y^2)dx = 0$.

- f. Find the scalar and vector projections of the vector $2\hat{i} - 3\hat{j} - 6\hat{k}$ on the line joining the points (3,4,-2) and (5,6,-3).
- g. Find $\frac{dy}{dx}$ if $x = \frac{2t}{1+t^2}$, $y = \frac{2t}{1-t^2}$.
- 3 i. If $\sqrt{1-x^6} + \sqrt{1-y^6} = k(x^3 - y^3)$, prove that $\frac{dy}{dx} = \frac{x^2}{y^2} \sqrt{\frac{1-y^6}{1-x^6}}$ 7
- ii. Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos^3 x}{x \sin 2x}$. 3
- 4 i. If $u = \tan^{-1}(x^2 + y^2 + z^2)$, show that $xu_x + yu_y + zu_z = \sin 2u$ 7
- ii. If sum of two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$. 3
- 5 i. Evaluate $\int \frac{2x+11}{\sqrt{x^2+10x+29}} dx$. 6
- ii. If $y = \tan^{-1} x$, prove that $(1+x^2)y_2 + 2xy_1 = 0$ 4
- 6 i. Solve the following differential equation $(1+y^2)dx = (\tan^{-1} y - x)dy$ 7
- ii. Find the derivative of $y = e^x$ by first principle. 3
- 7 i. In a triangle AOB , angle $AOB = 90^\circ$. If P, Q are the points of trisection of \overline{AB} , prove that $OP^2 + OQ^2 = \frac{5}{9} AB^2$ by vector method. 6
- ii. Evaluate $\int e^x \left(\frac{1}{x^2} - \frac{2}{x^3} \right) dx$. 4