LECTURE NOTES

ON

FLUID MECHANICS

4th SEMESTER MECHANICAL

BY

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MODEL SET QUESTION FOR PRACTICE

Sub:- FM (Fluid Mechanics) SET-1

Full Marks - 80 Time - 3 hours.

Answer any five questions including question No - 01 and 02.

① (a) Define specific weight and specific gravity. [2X10]

(b) Define pressure and state its unit.

(c) Define the term viscosity.

(d) Write Archimedes's principle.

(e) Define metacentre and metacentric height.

(f) What is the difference between laminar and turbulent flow?

(g) What is pitot tube?

(h) State Darcy's formula for loss of head in pipe?

(i) What do you mean by impact of jet?

(j) What is venacontracta?

(2) Answer any five. [5X6]

(i) With neat sketch explain the working of Bourdon's tube pressure gauge.

(ii) Derive an equation for the total pressure on a vertical immerged surface.

(iii) The diameter of a pipe at the section 1 and 2 are 10cm and 15cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section ① is 5m/s. Find the velocity at section 2.

(iv) Write down the expression of loss of energy due to friction according to Darcy's formula and Chezy's formula with proper notation.

(v) A sharp-edged orifice of 5 cm diameter discharges water under a head of 4.5 m. Determine the coefficient of discharge if the measured rate of flow is 0.0122 m^3/s.

(vi) Derive an expression for the force of jet on a fixed plate. |10

<3> A rectangular plane surface is 2 m wide and 3 m deep. It lies in vertical plane in water. Determine the total pressure and position of centre of pressure on the plane surface when its upper edge is horizontal and coincides with the water surface. Also find the total pressure and position of centre of pressure when the upper edge is 2.5 m below the free water surface.

<4> Describe the orifice coefficients and write down the relationship among them. |10

<5> Water flows through a pipe of 200mm in diameter and 60m long with a velocity of 2.5 m/s. Find the head lost due to friction using |10

(a) Darcy's formula, $f = 0.005$

(b) Chezy's formula $C = 55$.

<6> Derive Bernoulli's equation and state the practical application in venturimeter. |10

<7> A jet of water 40 mm diameter moving with a velocity of 120 m/s impinging on a series of vanes moving with a velocity of 5 m/sec. Find the force exerted, workdone and efficiency. |10

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All The Best.

MODEL SET QUESTION FOR PRACTICE

Sub:- Fluid Mechanics. [SET-2] Time - 3 hours

Full Marks:- 80

Answer any five questions including question No. 01 and 02.

① (i) Define density and state its unit. [2X10

(ii) Define pascal's law.

(iii) Define the term surface tension.

(iv) What is the function of piezometer?

(v) Define Bouyancy force?

(vi) What is the difference between compressible and incompressible fluid?

(vii) What are the assumptions taken in deriving the Bernoulli's equation.

(viii) What is chezy's constant?

(ix) Define hydraulic gradient?

(x) Define pressure head and velocity head.

[5X6

② (i) Explain absolute pressure, atmospheric pressure and gauge pressure and state their relations.

(ii) Explain the working and function of a pitot tube.

(iii) A simple U-tube manometer containing mercury is connected to a pipe in which fluid of specific gravity 0.8 and having vaccum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vaccum pressure in the pipe if the difference of mercury level in the two limbs is 40cm and the height of fluid in the left from the centre of pipe is 15cm. below.

(iv) Derive continuity equation.

(v) Water is flowing through a pipe 1500 m long ~~and~~ 400 ~~mm diameter~~ with a velocity of 0.7 m/s. What should be the diameter of pipe if the loss of head due to friction is 8.7 m. Take f for the pipe is 0.01.

(vi) Explain hydraulic gradient and total gradient line. 10

<3> Describe different types of manometers.

<4> (i) The head of water over an orifice of diameter 40 mm is 10 m. Find the actual discharge and actual velocity of jet at vena contracta. 5

take $C_d = 0.6$, $C_v = 0.98$.

<4> (ii) The discharge over a rectangular notch is 0.135 m^3/s when the water level is 22.5 m above the sill. If the coefficient of discharge is 0.6 find the length of notch. 5

<5> Derive an expression for the force of jet on a fixed and inclined plate. 10

<6> Describe different types of flows. 10

<7> Derive the expression of actual discharge in venturimeter and state its practical applications. 10

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All The Best.

NOTCHES AND WEIRS

Introduction :-

→ A notch is a device used for measuring the rate of flow of a liquid through a small channel or a tank.

→ It may be defined as an opening in the side of a tank or a small channel in such a way that the liquid surface in the tank or channel is below the top edge of the opening.

→ A weir is a concrete structure, placed in an open channel over which the flow occurs. It is generally in the form of vertical wall with a sharp edge at the top.

→ The notch is of small size while the weir is of a bigger size.

→ The notch is generally made of metallic plate while the weir is made of concrete structure.

Classification :-

The notches are classified as

① According to the shape of ~~notch~~ opening

(i) rectangular notch
(ii) Triangular notch
(iii) Trapezoidal notch
(iv) stepped notch.

② According to the effect of the sides of nappe :-

(i) Notch with end contraction
(ii) Notch without end contraction.

Weirs are classified according to shape

(a) According to the shape of opening

(i) Rectangular weir
(ii) Triangular weir
(iii) Trapezoidal weir.

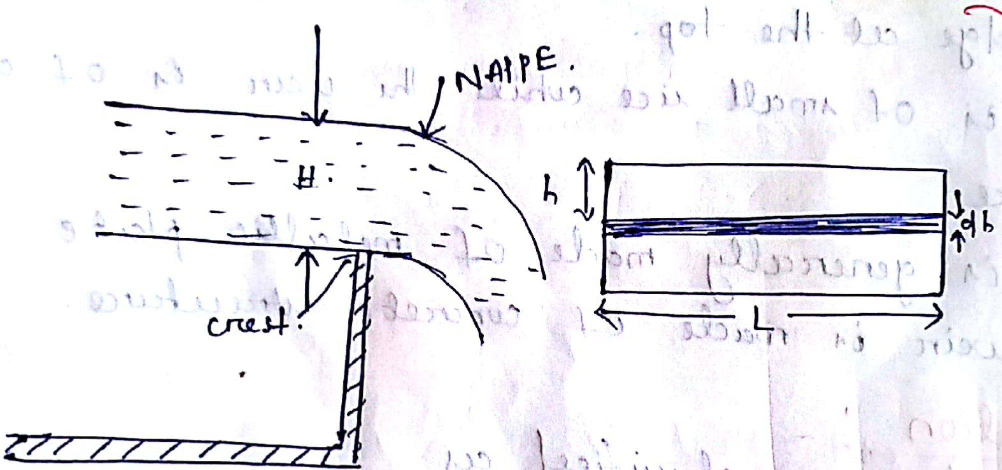
(b) According to the shape of crest

(i) Sharp-crested weir
(ii) Narrow-crested weir
(iii) Broad-crested weir
(iv) Ogee-shaped weir.

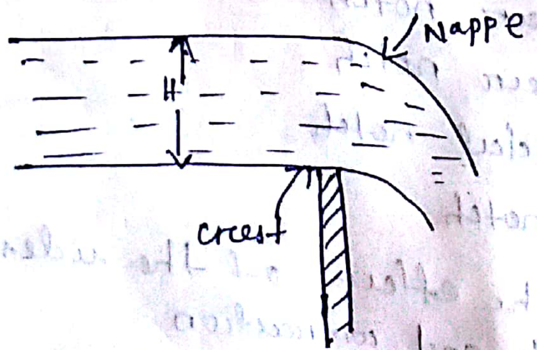
(c) According to the effect of sides on the emerging nappe:-

(i) weir with end contraction
(ii) weir without end contraction.

DISCHARGE OVER A RECTANGULAR NOTCH OR WEIR



Rectangular Notch



Rectangular Weir

consider a rectangular notch or weir provided in a channel carrying water.

H = head of water over the crest

L = Length of the notch or weir.

→ To find the discharge of water flowing over the weir or notch, consider an elementary horizontal strip of water of thickness dh and length L at a depth h from the free surface.

Area of Strip $= L \times dh$.

theoretical velocity of water flowing through strip $= \sqrt{2gh}$

The discharge dQ, through strip is

$$dQ = C_d \times \text{area of strip} \times \text{Theoretical velocity}$$

$$\boxed{dQ = C_d \times L \times dh \times \sqrt{2gh}}$$

$$Q = \int_0^H C_d \times L \times \sqrt{2gh} \times dh$$

$$= C_d \times L \times \sqrt{2g} \times \int_0^H h^{1/2} dh$$

$$= C_d \times L \times \sqrt{2g} \times \left[\frac{h^{1/2+1}}{1/2+1}\right]_0^H$$

$$= C_d \times L \times \sqrt{2g} \times \left[\frac{h^{3/2}}{3/2}\right]_0^H$$

$$= C_d \times L \times \sqrt{2g} \times \tfrac{2}{3} \times (H)^{3/2}$$

$$Q = C_d \times L \times \sqrt{2g} \times \tfrac{2}{3} \times (H)^{3/2}$$

$$\boxed{Q = \tfrac{2}{3} C_d L \sqrt{2g} \times (H)^{3/2}}$$

Q) Find the discharge of water flowing over a rectangular notch of 2m length when the constant head over the notch is 300mm. $C_d = 0.60$

Head over the notch $H = 300\,mm = 0.30\,m$

$L = 2\,m.$

$C_d = 0.60$

$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g} \times (H^{3/2})$$

$$= \frac{2}{3} \times 0.6 \times 2.0 \times \sqrt{2 \times 9.81} \times (0.30)^{3/2}$$

$$\boxed{Q = 0.582\,m^3/s}$$

Q) Determine the height of a rectangular weir of length 6m to ~~H~~ be built across a rectangular channel. The maximum length of water on the upstream side of the weir is 1.8 m and discharge is 2000 litre/s. Take $C_d = 0.6$

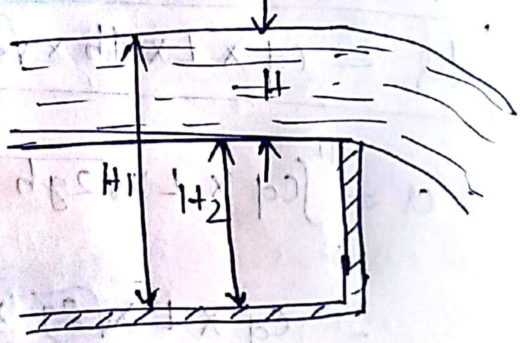
$L = 6m.$

$H_1 = 1.8\ m$

$Q = 2000\ lt/s.$

$= 2m^3/s.$

$C_d = 0.6$



$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g} \times H^{3/2}$$

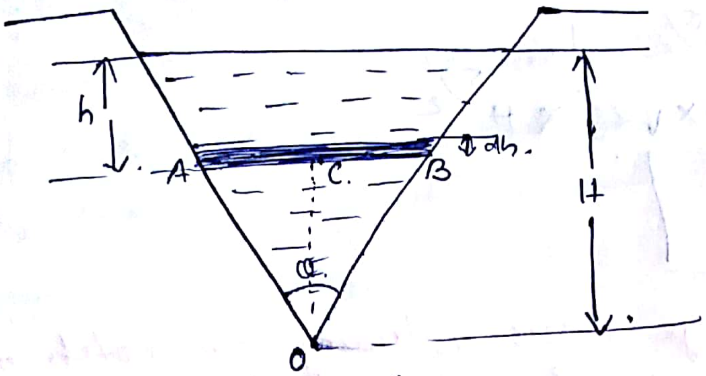
$$\Rightarrow 2 = \frac{2}{3} \times 0.6 \times 6.0 \times \sqrt{2 \times 9.81} \times H^{3/2}$$

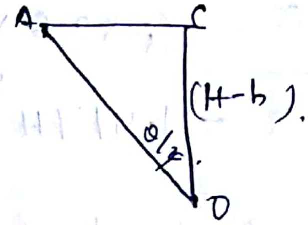
$$\Rightarrow H^{3/2} = \frac{2.0}{10.623}$$

$$\boxed{H = 0.328\,m}$$

$$\boxed{\begin{aligned} H_2 &= H_1 - H \\ &= 1.8 - 0.328 = 1.472\,m. \end{aligned}}$$ (Ans)

DISCHARGE OVER A TRIANGULAR NOTCH OR WEIR





H = head of water above the V-notch.
θ = angle of notch.
consider the horizontal strip of water of thickness 'dh' at a depth of h from the free surface of water.

$$\tan \theta/2 = \frac{AC}{OC} = \frac{AC}{(H-h)}$$

$$AC = (H-h) \tan \theta/2$$

$$AB = \text{width of strip} = 2 \times AC$$

$$= 2 \times (H-h) \tan \theta/2 \times dh$$

theoretical velocity of water through strip $= \sqrt{2gh}$

Discharge through the strip

$$dQ = C_d \times \text{Area of strip} \times \text{velocity}$$

$$= C_d \times 2(H-h) \tan \theta/2 \times dh \times \sqrt{2gh}$$

$$= 2 \times C_d \times (H-h) \tan \theta/2 \times \sqrt{2gh} \times dh$$

$$Q = \int_0^H 2 C_d \times (H-h) \tan \theta/2 \times \sqrt{2gh} \times dh$$

$$= 2 C_d \times \tan \theta/2 \times \sqrt{2g} \times \int_0^H (H-h)\, h^{1/2}\, dh$$

$$= 2 \times C_d \times \tan \theta/2 \times \sqrt{2g} \left[\int_0^H H h^{1/2}\, dh - \int_0^H h^{3/2}\, dh \right]$$

$$= 2 \times C_d \times \tan \theta/2 \times \sqrt{2g} \left[\tfrac{2}{3} H^{5/2} - \tfrac{2}{5} H^{5/2} \right]$$

$$= 2 \times C_d \times \tan \theta/2 \times \sqrt{2g} \times \left[\tfrac{4}{15} H^{5/2} \right].$$

$$\boxed{Q = \frac{8}{15} C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}}$$

for a V-notch $C_d = 0.6$

$\theta = 90^\circ$, $\tan \theta/2 = 1$.

$$Q = \frac{8}{15} \times 0.6 \times 1 \times \sqrt{2g} \times H^{5/2}$$

$$\boxed{Q = 1.417 H^{5/2}}$$

Q) Find the discharge over a triangular notch of angle 60° when the head over the V-notch is 0.3 m. $C_d = 0.6$.

Ans $\theta = 60^\circ$

$H = 0.3$ m.

$C_d = 0.6$

$$Q = \frac{8}{15} \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$= \frac{8}{15} \times 0.6 \times \tan 30^\circ \times \sqrt{2 \times 9.81} \times (0.3)^{5/2}$$

$Q = 0.040\ m^3/s$. (Ans)

FLOW THROUGH PIPES (CHAPTER-6)

Loss of energy in pipe: –

when a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as follows:

Energy loss

- **Major energy loss**
  - This is due to friction.
  - (a) Darcy – Weisbach formula.
  - (b) chezy's Formula.
- **Minor energy loss**
  - (a) Sudden expansion of pipe
  - (b) Sudden contraction of pipe
  - (c) Bend in pipe.
  - (d) pipe fittings
  - (e) obstruction in pipe.

(I) Loss of energy due to friction.

(a) Darcy – Weisbach Formula: –

This loss of energy in pipes due to friction is calculated from Darcy – Weisbach equation.

$$\boxed{h_f = \frac{4fLV^2}{2g}}$$

h_f = Loss of head due to friction.

f = Coefficient of friction $= \frac{16}{Re}$

$$f = \frac{16}{Re} \quad (Re < 2000)$$

$$f = \frac{0.079}{Re^{1/4}} \quad (Re\ (4000 - 10^6))$$

L = Length of pipe.

V = mean velocity of flow.

d = diameter of pipe.

(b) chezy's formula

The expression for loss of head due to friction

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2$$

h_f = loss of head due to friction

A = area of cross-section of pipe

p = wetted perimeter of pipe

v = Mean velocity of flow.

p = perimeter of pipe.

L = Length of pipe.

$\frac{A}{P} = \frac{\text{Area of flow}}{\text{perimeter}}$ is called hydraulic mean depth

$\frac{A}{P}$ = (hydraulic mean depth or hydraulic radius)

(A/p) is denoted by 'm'.

∴ hydraulic mean depth $m = A/P = \frac{\pi/4\, d^2}{\pi d} (= d/4)$.

$\frac{A}{P} = m$ or $\left(\frac{P}{A}\right) = 1/m$.

$$h_f = \frac{f'}{\rho g} \times L \times V^2 \times \frac{1}{m}$$

$$\Rightarrow V^2 = h_f \times \left(\frac{\rho g}{f'}\right) \times m \times \frac{1}{L}$$

$$\Rightarrow V = \sqrt{\frac{\rho g}{f'} \times m \times \left(\frac{h_f}{L}\right)}$$

$$\boxed{V = \sqrt{\frac{\rho g}{f'}} \times \sqrt{m \frac{h_f}{L}}}$$

where $\sqrt{\frac{\rho g}{f'}} = C$ (C = chezy's constant)

$$\frac{h_f}{L} = i$$

$$\therefore \boxed{V = C \times \sqrt{m \times i}}$$

This is known as chezy's formula.

$\boxed{m = d/4}$

Q) Find the head lost due to friction in a pipe of diameter 300mm and length 50 m through which water is flowing at a velocity of 3m/s. using

(i) Darcy's formula [Data $\nu = 0.01$ stoke)

(ii) chezy's formula.

$d = 300mm = 0.30m.$

$L = 50m.$ $\qquad \nu = 0.01$ stoke $= 0.01 \times 10^{-4}\, m^2/s$.

$v = 3m/s.$

$C = 60.$

$$Re = \frac{Vd}{\nu} = \frac{3 \times 0.30}{0.01 \times 10^{-4}} = 9 \times 10^5.$$

$$f = \frac{0.079}{Re^{1/4}} = \frac{0.079}{(9 \times 10^5)^{1/4}} = 0.00256.$$

(i) $$h_f = \frac{4 \times f \times L \times v^2}{d \times 2g} \cdot \text{(} \cancel{\text{Chezy's}} \text{ Darcy's formula)}.$$

$$= \frac{4 \times 0.00256 \times 50 \times 3^2}{0.3 \times 2 \times 9.81}$$

$$\boxed{h_f = 0.7828\, m \text{ (Ans)}}$$

(ii) Chezy's formula.

$$V = C\sqrt{mi}$$

$$C = 60., \quad m = d/4 = \frac{0.3}{4} = 0.075\, m.$$

$$V = C \times \sqrt{mi}$$

$$\Rightarrow 3 = 60 \times \sqrt{0.075 \times \frac{h_f}{L}}$$

$$\Rightarrow \left(\frac{3}{60}\right)^2 = 0.075 \times \frac{h_f}{L}$$

$$\Rightarrow \frac{h_f}{L} = \left(\frac{3}{60}\right)^2 \times \frac{1}{0.075}$$

$$\Rightarrow h_f = \left(\frac{3}{60}\right)^2 \times \frac{1}{0.075} \times 50 = 1.665\, m. \text{ (Ans)}$$

Q) Find the diameter of pipe of length 2000 m when the rate of flow of water through the pipe is 200 liters/sec and head loss due to friction is 4m.

$\boxed{C = 50}$

Ans $L = 2000 m.$

$Q = 2000 \text{ litre/s} = 0.2 \ m^3/s.$

$h_f = 4m.$

$C = 50.$

$$V = \frac{\text{discharge}}{\text{Area}} = \frac{0.2}{(\frac{\pi}{4}d^2)}$$

$$V = C \times \sqrt{mi}$$

$$= 50 \times \sqrt{\frac{d}{4} \times \frac{h_f}{L}}$$

$$\frac{0.2 \times 4}{\pi/4 \ d^2} = 50 \times \sqrt{\frac{d}{4} \times \frac{4}{2000}}$$

$$\Rightarrow \frac{0.2 \times 4}{\pi/4 \ d^2 \times 50} = \sqrt{\frac{d}{\cancel{4}} \times \frac{\cancel{4}}{2000}}$$

$$\Rightarrow \left(\frac{0.2 \times 4}{\pi d^2 \times 50}\right)^2 = \frac{d}{2000}$$

$$\Rightarrow \frac{(0.2)^2 \times (4)^2}{\pi^2 \times d^4 \times (50)^2} = \frac{d}{2000}$$

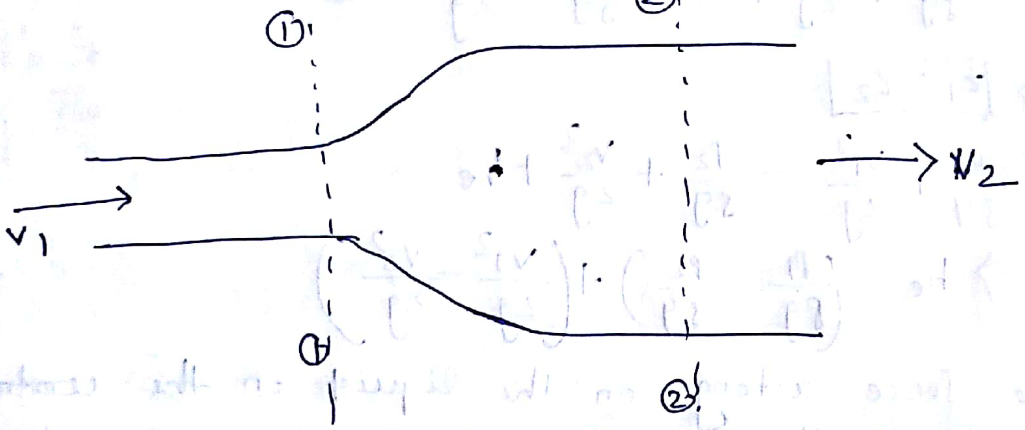
$$\Rightarrow \frac{(0.2)^2 \times 16 \times 2000}{\pi^2 \times (50)^2} = d^5$$

$$\Rightarrow d = \sqrt[5]{0.0518} = 0.553 m = 553 mm \text{ (Ans)}$$

Minor Energy Losses :-

The loss of energy due to friction in pipe is known as major loss while the loss of energy due to change of velocity of the ~~fo~~ fluid is called minor loss of energy.

① Loss of head due to sudden enlargement



Consider a liquid flowing through a pipe which has sudden enlargement as shown in above figure. Consider two sections ①-① and ②-② before and after enlargement.

P_1 = pressure intensity at section ①-①

v_1 = velocity of flow at section ①-①

a_1 = area of pipe at section ①-①

P_2 = pressure intensity at section ②-②

V_2 = velocity of flow at section ②-②

a_2 = area of pipe at section ②-②

→ Due to sudden change in diameter of pipe from D_1 to D_2, the liquid flowing from the smaller pipe is not able to follow the change of boundary. Thus the flow separates from the boundary and turbulent eddies are formed.

The loss of energy takes place due to formation of these eddies.

p' = pressure intensity of the liquid eddies.

h_e = Loss of head due to sudden enlargement.

Applying Bernoulli's equation

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{head loss}$$

$\Rightarrow \boxed{z_1 = z_2}$

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_e$$

$$\Rightarrow h_e = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)$$

→ The force acting on the liquid in the control volume in the direction of flow is given by

$$\boxed{F_x = p_1 A_1 + p'(A_2 - A_1) - p_2 A_2}$$

→ $p' = p_1$

$$F_x = p_1 A_1 + p_1(A_2 - A_1) - p_2 A_2 = p_1 A_2 - p_2 A_2$$

$$\boxed{F_x = A_2(p_1 - p_2)}.$$

Momentum of liquid in section 1-1 $= \rho_1 A_1 V_1^2$.

momentum of liquid at section 2-2 $= \rho A_2 V_2^2$

change in momentum $= \rho A_2 V_2^2 - \rho A_1 V_1^2$

continuity equation $\boxed{A_1 V_1 = A_2 V_2}$

$$\boxed{A_1 = \frac{A_2 V_2}{V_1}}$$

Change in momentum/sec $= \rho A_2 V_2^2 - \rho \times A_1 V_1^2$

$$= \rho A_2 V_2^2 - \rho \times \frac{A_2 V_2}{V_1} \times V_2^2$$

$$= \rho A_2 V_2^2 - \rho A_2 V_1 V_2$$

$$= \rho A_2 (V_2^2 - V_1 V_2).$$

Net force acting on control volume in the direction of flow must be equal to the rate of change of momentum.

$$(P_1 - P_2) A_2 = \rho A_2 (V_2^2 - V_1 V_2)$$

$$\Rightarrow \frac{P_2 - P_2}{\rho} = V_2^2 - V_1 V_2$$

$$\Rightarrow \boxed{\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1 V_2}{g}}$$

$$\therefore h_e = \left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)$$

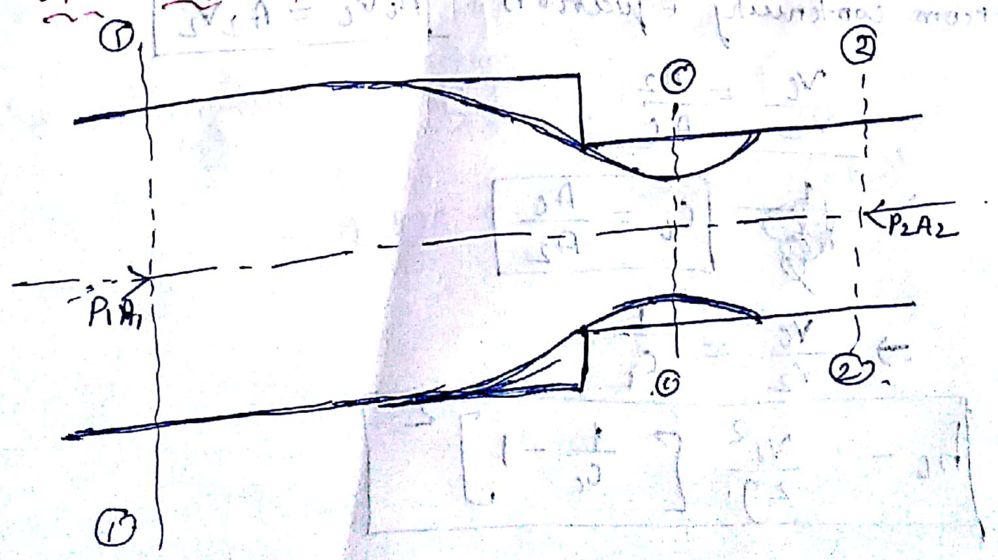
$$= \frac{V_2^2 - V_1 V_2}{g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

$$= \frac{2V_2^2 - 2V_1 V_2 + V_1^2 - V_2^2}{2g}$$

$$= \frac{V_2^2 + V_1^2 - 2V_1 V_2}{2g}$$

$$\boxed{h_e = \frac{(V_1 - V_2)^2}{2g}}$$

Loss of Head due to sudden Contraction :-



(P)

→ Consider a liquid flowing in a pipe which has a sudden contraction in area as shown in fig.

→ consider two section (1-1) and (2-2) before and after contraction.

→ As the liquid goes from a large pipe to a small pipe, the area of flow goes on decreasing and becomes minimum at section (C-C). This section is called as vena contracta.

→ After section C-C, a sudden enlargement takes place. The loss of head due to sudden contraction is actually due to sudden enlargement from vena contracta to smaller pipe.

Let A_c = Area of flow at section C-C.

V_c = velocity of flow at section C-C.

A_2 = Area of flow at section 2-2.

V_2 = velocity of flow at section 2-2.

h_c = Loss of head due to sudden contraction.

$$h_c = \frac{(V_c - V_2)^2}{2g}$$

$$= \frac{V_2^2}{2g}\left[\frac{V_c}{V_2} - 1\right]^2$$

from continuity equation $\boxed{A_c V_c = A_2 V_2}$

$$\frac{V_c}{V_2} = \frac{A_2}{A_c}$$

$$\Rightarrow \boxed{C_c = \frac{A_c}{A_2}}$$

$$\Rightarrow \frac{V_c}{V_2} = \frac{1}{C_c}$$

$$\boxed{h_c = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2}$$

where $K = \left(\frac{1}{C_c} - 1\right)^2$

$$\boxed{h_c = \frac{K V_2^2}{2g}}$$

$C_c = 0.62$

$\therefore K = \left(\frac{1}{0.62} - 1\right)^2 = 0.375$

$$\boxed{h_c = 0.375 \frac{V_2^2}{2g}}$$

If the C_c value is not given then

$$\boxed{h_c = 0.5 \frac{V_2^2}{2g}}$$

Q) Find the loss of head when the pipe of diameter 200mm is suddenly enlarged to a diameter of 400mm. The rate of flow of water through the pipe is 250 litre/sec.

$D_1 = 200mm = 0.2m$

$D_2 = 400mm = 0.4m.$

$A_1 = \pi/4 \, D_1^2 = \pi/4 \times (0.2)^2 = 0.03141 m^2$

$A_2 = \pi/4 \, D_2^2 = \pi/4 \times (0.4)^2 = 0.12564 m^2$

$Q = 250 litre/s = 0.25 m^3/s.$

$V_1 = Q/A_1 = 7.96 m/s.$

$V_2 = Q/A_2 = 1.99 m/s.$

$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2 \times 9.81}$

$= 1.816$ m of water. (Ans)

3) Loss of Head at the Entrance of pipe.

This is the loss of energy which occurs when a liquid enters a pipe which is connected to large tank.

$$\boxed{h_i = 0.5\frac{v^2}{2g}}$$

v = velocity of liquid in pipe.

4) Loss of Head at the Exit of pipe :-

This is loss of head due to velocity of liquid at the outlet of pipe. It is denoted as h_o

$$\boxed{h_o = \frac{v^2}{2g}}$$

v = velocity of liquid at outlet of pipe.

5) Loss of head due to Bend in pipe :-

When there is bend in pipe, the velocity of flow changes due to which formation of eddies takes place.

$$\boxed{h_b = \frac{Kv^2}{2g}}$$

h_b = loss of head due to bend.
v = velocity of flow.
K = coefficient of bend.

6) Loss of Head in various pipe fittings :-

This is the loss of head in various pipe fittings. It is expressed as

$$\boxed{\frac{Kv^2}{2g}}$$

V = velocity of flow.
K = coefficient of pipe fitt.

HYDRAULIC GRADIENT LINE :-

It is defined as the line which gives the sum of pressure head (P/w) and datum head (z) of a flowing fluid in a pipe with respect to some reference line.

→ It is briefly written as H.G.L (Hydraulic gradient Line)

TOTAL ENERGY LINE :-

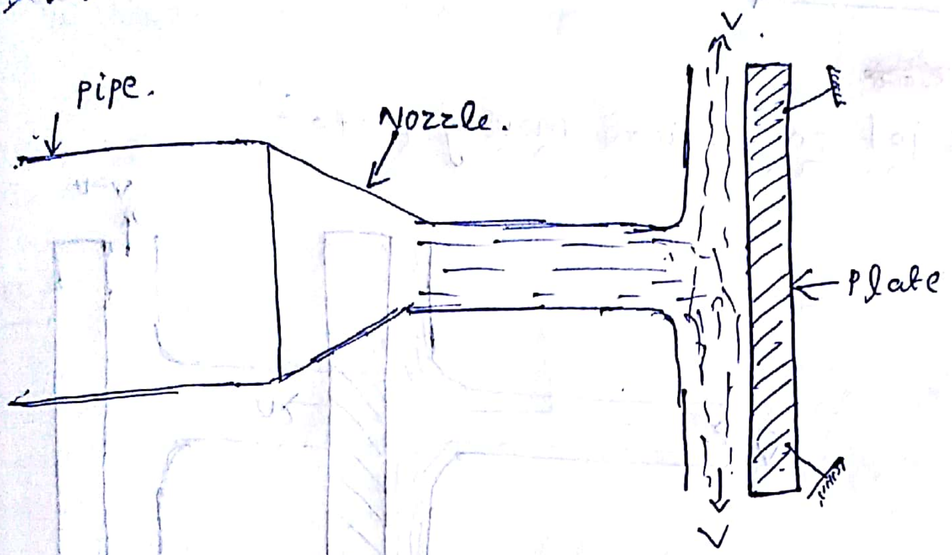
It is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line.

→ It is briefly written as T.E.L (Total Energy Line).

— 0 —

IMPACT OF JET (CHAPTER-7)

Impact of jet on a fixed vertical flat plate



→ consider a jet of water coming out from the nozzle, strikes a flat vertical plate.

V = velocity of the jet.

d = diameter of the jet

a = area of crossection of jet $= \frac{\pi}{4} d^2$

→ The jet after striking the plate, will move along the plate. But the plate is at right angles to the jet. Hence the jet after striking will get deflected through 90°.

→ After striking the component of the velocity of jet in the direction of jet is zero.

The force exerted by the jet on the plate in the direction of jet

F_x = Rate of change of momentum in direction of force

$$= \frac{\text{Initial momentum} - \text{final momentum}}{\text{Time}}$$

$$= \frac{(\text{Mass} \times \text{initial velocity}) - (\text{Mass} \times \text{Final velocity})}{\text{Time}}$$

$$= \frac{\text{Mass}}{\text{Time}} (\text{initial velocity} - \text{Final velocity})$$

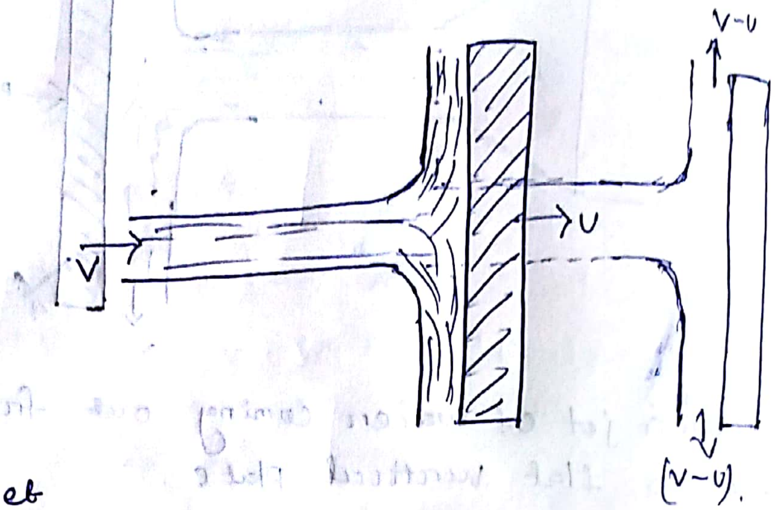
$$= \frac{\text{Mass}}{\text{Time}} (V - 0)$$

$$F_x = \rho a V (V-0). \quad \left[\text{Mass/sec} = \rho \times aV\right]$$

$$\boxed{F_x = \rho a V^2}$$

~~Force on Plate~~

Impact of jet on vertical moving plate :-



→ V = velocity of jet
a = area of cross-section of the jet
u = velocity of flat plate.

→ In this case, the jet does not strike the plate with a velocity V, but it strikes with a relative velocity.

→ The relative velocity is equal to the difference of absolute velocity of jet of water and the velocity of plate.

→ The relative velocity = $(V-u)$

→ Mass of water striking the plate per sec = $\rho \times$ Area of jet $\times$ velocity with which jet strikes the plate

$$= \rho a (V-u).$$

→ Force exerted by the jet on the moving plate in the direction of jet

F_x = Mass of water × (initial velocity - final velocity)

$$= \rho a (V-u) \left[(V-u) - 0\right].$$

$$\boxed{F_x = \rho a (V-u)^2}$$

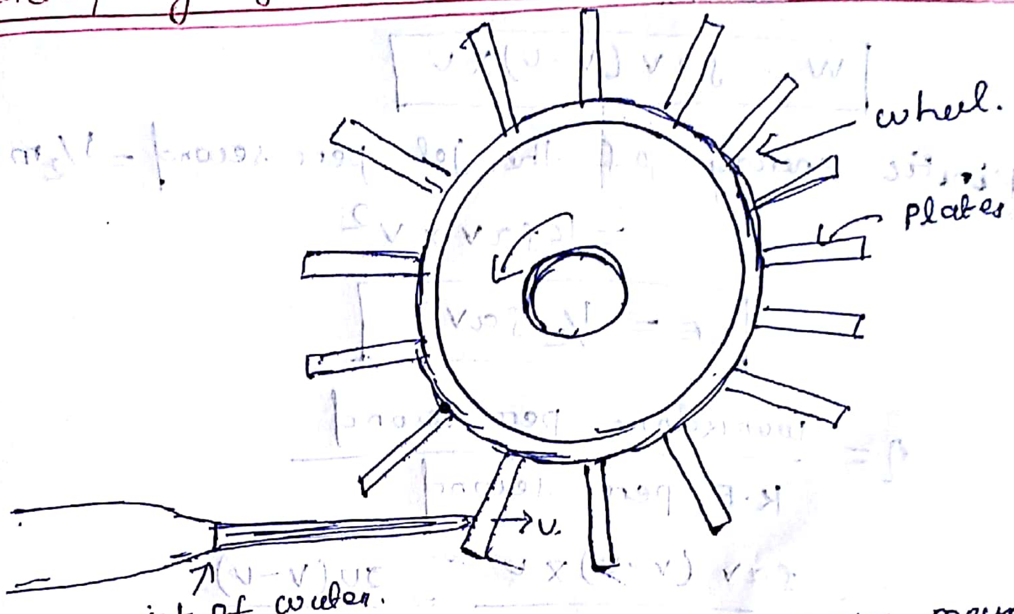
→ The work will be done by the jet on the plate, as plate is moving.

workdone = Force × velocity

$= F_x \times U$

$$\boxed{W = \rho a (V-U)^2 \times U}$$

Force exerted by a jet of water on a series of vanes



→ In actual practice, a large number of plates are mounted on the circumference of a wheel at a fixed distance apart.

→ The jet strikes a plate and due to the force exerted by the jet on the plate, the wheel starts moving.

V = velocity of jet.

d = diameter of jet.

a = cross-sectional area of jet $= \pi/4\, d^2$

U = velocity of vane.

→ Mass of water per second striking the series of plates $= \rho a V$.

→ Jet strikes the plate with a velocity $= (V-U)$

→ The force exerted by the jet in the direction of motion of plate

F_x = Mass ~~of~~ × (initial velocity − final velocity)

$= \rho a V[(V-U)-0]$

$$\boxed{F_x = \rho a V(V-U)}$$

workdone = Force × ~~Distance~~ velocity

$= F_x \times U$

$$\boxed{W = \rho a V(V-U) \times U}$$

Kinetic energy of the jet per second $= \frac{1}{2} m V^2$

$= \frac{1}{2} \rho a V \times V^2$

$$\boxed{KE = \frac{1}{2} \rho a V^3}$$

$$\eta = \frac{\text{workdone per second}}{\text{K.E per second}}$$

$$= \frac{\rho a V(V-U) \times U}{\frac{1}{2} \rho a V^3} = \frac{2U(V-U)}{V^2}$$

$$\boxed{\eta = \frac{2U(V-U)}{V^2}}$$

condition for Maximum Efficiency :-

$$\frac{d\eta}{dU} = 0$$

$$\Rightarrow \frac{d}{dU}\left[\left(\frac{2U(V-U)}{V^2}\right)\right] = 0$$

$$\Rightarrow \frac{d}{dU}\left(\frac{2UV - 2U^2}{V^2}\right) = 0$$

$$\Rightarrow \frac{2V - 2 \times 2U}{V^2} = 0$$

$$\Rightarrow 2V - 4U = 0$$

$$\Rightarrow V = 2U \Rightarrow \boxed{U = V/2}$$

Maximum efficiency

$$\eta_{max} = \frac{2u(v-u)}{v^2}$$

$$= \frac{2u(2u-u)}{(2u)^2}$$

$$= \frac{2u \times u}{4u^2} = 1/2 = 50\%$$

$$\boxed{\eta_{max} = 50\%}$$

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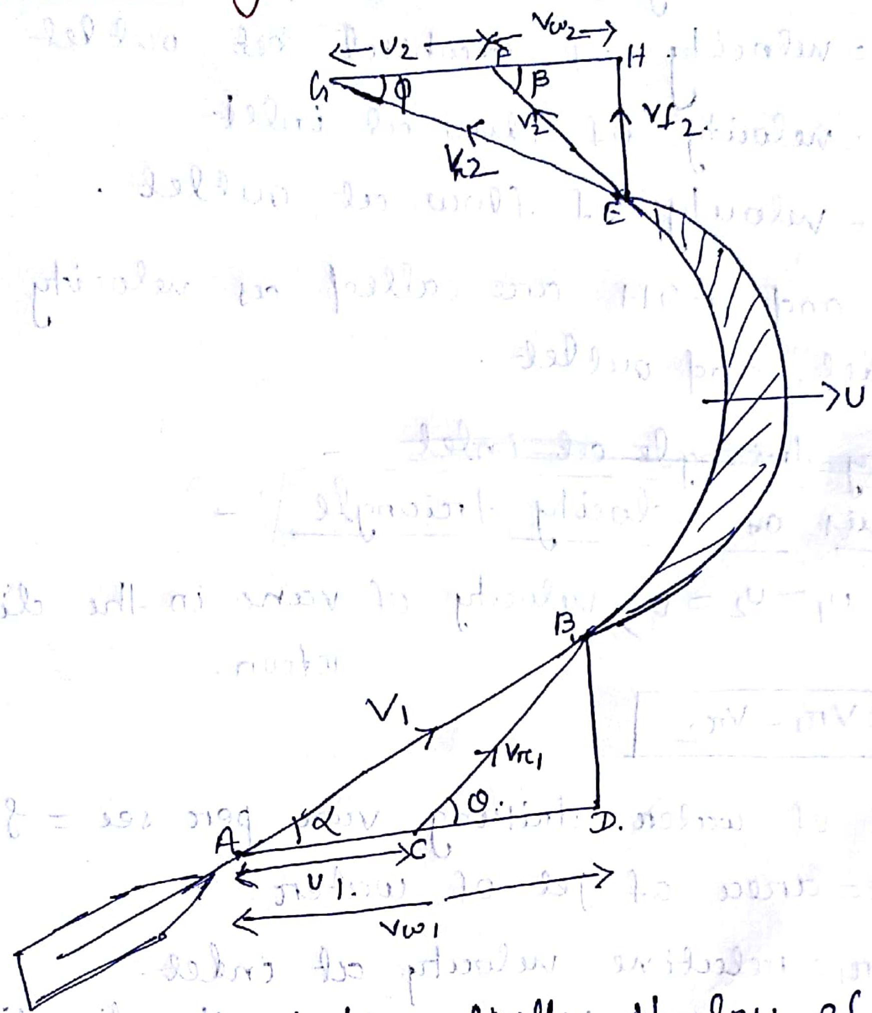
Impact on a moving curved plate :-



→ As the jet strikes tangentially, the loss of energy due to impact of the jet will be zero. as the plate is moving, the velocity with which jet of water strikes is equal to the relative velocity of the jet with respect to the plate.

V_1 = velocity of the jet at inlet
U_1 = velocity of vane at inlet
V_{r_1} = relative velocity of jet and plate at inlet.

α = blade angle (inlet)
θ = vane angle (inlet).
V_2 = velocity of jet at outlet.
U_2 = velocity of vane at outlet.
V_{r_2} = relative velocity of jet at outlet.
β = blade angle at outlet.
ϕ = vane angle at outlet.

V_{w_1} = velocity of whirl at inlet
V_{w_2} = velocity of whirl at outlet
V_{f_1} = velocity of flow at inlet
V_{f_2} = velocity of flow at outlet.

ABD and EGH are called as velocity triangles at inlet and outlet.

~~velocity triangle at inlet~~ :-

Analysis of velocity triangle :-

$U_1 = U_2 = U$ = velocity of vane in the direction of motion.

$$\boxed{V_{r_1} = V_{r_2}}$$

→ Mass of water striking vane per sec = $\rho a V_{r_1}$

a = area of jet of water.

V_{r_1} = relative velocity at inlet.

→ Force exerted by the jet in the direction of moti

$$\boxed{F_x = \rho a V_{r_1} (V_{w_1} + V_{w_2})}$$

If $\beta = 90^\circ$, $V_{w_2} = 0$

$$\boxed{F_x = \rho a V_{r_1} \times V_{w_1}}$$

→ β is obtus angle

$$\boxed{F_x = \rho a V_{r_1} [V_{w_1} - V_{w_2}]}$$

Thus in general F_x can be written as

$$\boxed{F_x = \rho a V_{r_1} [V_{w_1} \pm V_{w_2}]}$$

→ workdone per second on the vane by jet

$= F_x \times U$

$$\boxed{W = \rho a V_{r_1} [V_{w_1} \pm V_{w_2}] \times U}$$

efficiency of jet :-

$$\eta = \frac{\text{output}}{\text{Input}}$$

$$= \frac{\text{workdone per second on the vane}}{K.E}$$

$$= \frac{\rho a V_{r_1} (V_{w_1} \pm V_{w_2}) \times U}{\frac{1}{2} m V_1^2}$$

$$= \frac{\rho \cdot a V_{r_1} (V_{w_1} \pm V_{w_2}) \times U}{\frac{1}{2} \times \rho a V_{r_1} \times V_1^2}$$

$$= \frac{(V_{w_1} \pm V_{w_2}) \times U}{\frac{1}{2} \times V_1^2}$$

$$\boxed{\eta = \frac{2U(V_{w_1} \pm V_{w_2})}{V_1^2}}$$

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